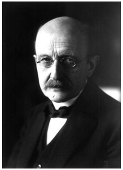
The Appearance of Events in Quantum Mechanics – a New Dynamical Law of Nature

"The interpretation of quantum mechanics (QM) has been dealt with by many authors, and I do not want to discuss it here. I want to deal with more fundamental things" – P.A.M. Dirac

Text-book QM resulted from the profound discoveries of:



M. Planck

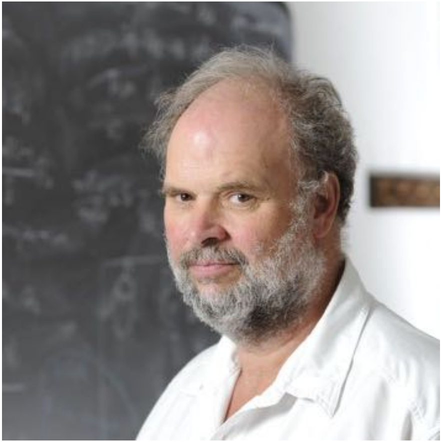
A. Einstein

W. Heisenberg

P. A. M. Dirac

Laws of Nature – J.F., December 7, 2021

Dedicated to the memory of *Vaughan Jones*, an outstanding mathematician deeply inspired by and inspiring Quantum Physics, and a very generous colleague; ...



Vaughan F. R. Jones, 31.12.1952 – 6.9.2020

... and of *Detlef Dürr*, who had a profound understanding of the foundations of Quantum Theory that he gladly shared with his colleagues

Detlef Dürr, 4.3.1951 – 3.1.2021, singing the "Lied vom Meer"

We have lost two great colleagues and wonderful friends who will be missed!

The Problem of the Big Unsolved Problems

To our dismay, it appears that, during the past 100 or more years, humanity has been unable to solve, or unwilling to cope with, any of the major problems threatening its own survival. And it is getting ever worse!

Examples of major problems not resolved, as of now:

- Nuclear disarmament - problem present for the past 75 years
- The demographic time-bomb - problem known for $\geq$ 75 years
- Climate change - problem identified $\geq$ 100 years ago
- Safe generation of clean and renewable energy, and energy storage
- Excesses of turbo-capitalism & of a dysfunctional monetary system
- Fostering of secular, enlightened, liberal societies; integration of immigrants from other cultural backgrounds into our societies
- Equal rights and equal privileges for women
- Arab-Israeli conflict, conflicts in Northern Ireland, Catalonia ...
- Unsolved problems in **Physics** and other sciences – Example: *"It seems clear that the present quantum mechanics is not in its final form."* (P.A.M. Dirac) *Completion of QM,* not Interpretation!

Contents and Credits

Contents:

1. What this lecture is about
2. The *ETH*-Approach to NR Quantum Mechanics (*QM*)
3. Huygens' Principle and the *Principle of Diminishing Potentialities*
4. Results for Limiting Models, as the speed of light $c \to \infty$

Summary and conclusions

Credits:

I wish to thank my collaborators and in particular my last PhD student *Baptiste Schubnel* and my friend *Alessandro Pizzo* for the joy of joint efforts on projects related to this lecture, as well as many colleagues at numerous institutions for instructive (and sometimes rather controversial) discussions on *QM*. Ideas of the late *Rudolf Haag* have proven fruitful in my work. – I am grateful to *the organizers* of this series for giving me an opportunity to present ideas and results – alas, not widely appreciated in the community, yet – on problems that are undoubtedly fundamental.

1. What this Lecture is About

Summary: Our purpose is to extend the standard formalism of *QM* and complete it in such a way that the resulting theory *makes sense*. The extension, yielding a *new Law of Nature*, is called

ETH - **Approach** to *QM*.

The *ETH* - Approach to *QM* supplies the fourth one of *four pillars* *QM* rests upon: (i) Physical quantities characteristic of a physical system are represented by s.-a. linear operators; (ii) the time evolution of operators representing physical quantities is given by the *Heisenberg equations*; (iii) meaningful notions of *Potential* and *Actual Events* and of *states*; and

(iv) a general statistical Law for the Time Evolution of states.

Core of talk: Besides sketching the *ETH*-Approach to *QM*, I will discuss simple models of a very heavy atom coupled to the radiation field in a limit where the speed of light tends to ∞, illustrating the *ETH*-Approach.

General goal: I am determined to remove some of the **enormous confusion** befuddling many colleagues who claim to work on the foundations of QM... Of course, hardly anybody expects that I will succeed – I do!

Topics to be Addressed – Today or in the Future

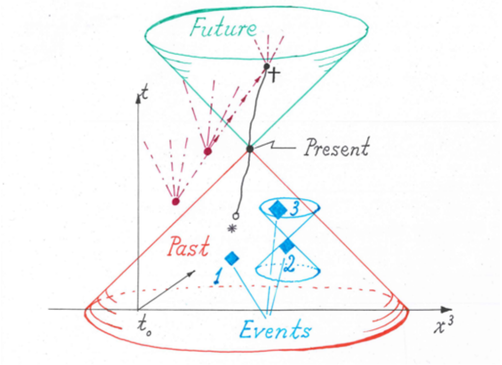
- Sketch a **novel completion** of Quantum Mechanics (*QM*), called *"ETH-Approach* to *QM*" – introducing:
- Sharp notions of: • *Isolated* physical systems in *QM* • *States* of physical systems ($\nearrow$ Gleason, Maeda) • *Potential -* and *Actual Events* in *QM* ($\nearrow$ Haag) – and discussing the:
- **Inadequacy of Schrödinger Eq.** $\rightarrow$ *ETH*-Approach predicts *statistical law* for time evolution of *states* of physical systems, confirming **probabilistic nature of** *QM* $\rightarrow$ *"quantum branching processes."*
- *Clarify the role of *(Space-)Time* and of the electromagnetic field in *QM*. How does *QM* distinguish between *past* and *future*? *Aristotle*: potentialities versus actualities; *Leibniz*' vision of space-time; Huygens' Principle. – The *ETH* - Approach predicts that a consistent *Quantum Theory of Events* is necessarily *relativistic*, and space-time must be *even- dimensional* ...

Today's focus: **Non-Relativistic QM**; but features of *local relativistic QT* are used to motivate some constructions; in particular, our models illustrating *ETH*-Approach to *QM* are inspired by *relativistic QED* !

The discussion of these models is among the main topics of lecture.

Generalities - 1: Theories Cannot be Fully Predictive

This drawing illustrates that relativistic theories are never fully predictive, and that there is a fundamental dichotomoy between past and future:



t_0: time right after inflation $\rightarrow$ event horizon $\Rightarrow$ initial conditions not fully accessible!

Past = History of Actualities (Facts) / Future = Ensemble of Potentialities

This fundamental dichotomy should be retained in Quantum Mechanics!

Generalities - 2: Direct versus Indirect Measurements

Besides leading to a precise law for the *time evolution of states* in *QM*, a key purpose of the *ETH*- Approach is to solve the infamous **"measurement problem"** of *QM*; namely the problem of developing a theory of *direct/projective measurements* highlighting the role of the e.m. field and free of internal inconsistencies. – [Will not treat *th. of indirect/weak measurements*, which is well understood, *assuming* that $\exists$ good theory of *direct/projective measurements* of probes! ($\nearrow$ Kraus et al.)

Examples of indirect measnts.: Haroche-Raimond exp.; particle tracks ...
(Mass-Küm, Ba-Be, BFFSch, BCrFFSch, ...; Fi-Te, BBenFF, BenFF, ...)]
$\downarrow$

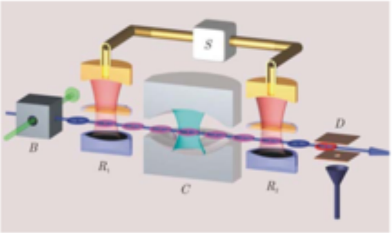
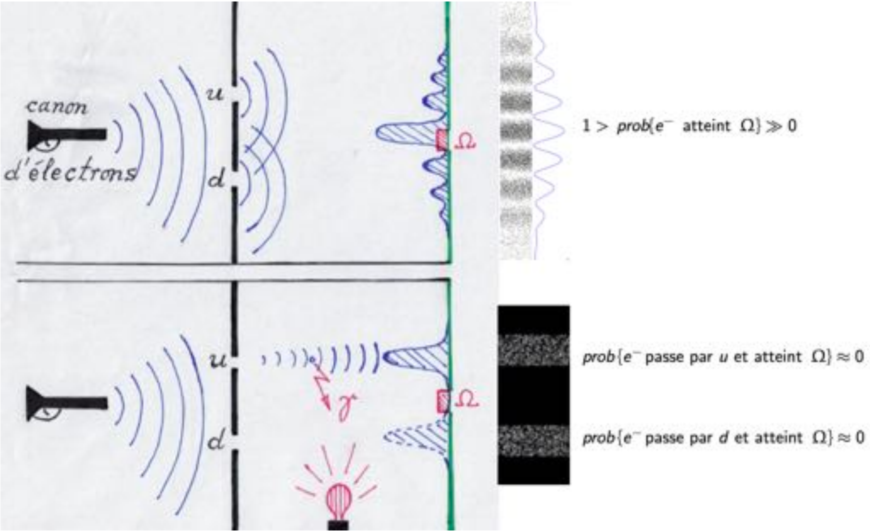


Fig. 4: Experimental setup to study microwave field states with the help of circular Rydberg atoms (see text).

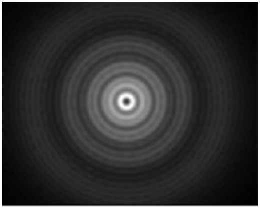
Generalities - 3: Waves or particles? Waves **and** particles!

Let's briefly discuss the well known double-slit experiment:



The role of light in the emergence of a classical world

Apparently, an *illumination* by light of the cavity to the right of the screen containing the double slit has the effect that *"electron waves"* are converted to *"corpuscles"* by eliminating interference effects (which also illustrates the *"retarded choice paradox"* of *Wheeler*): The quantum world, which involves *potentialities*, rep. by non-commuting ops. giving rise to *interference* and to Heisenberg's uncertainty relations, approaches the *factual classical world!* – This becomes strikingly manifest by considering a spherically symmetric ball of radioactive material emitting α-particles into a cavity (*Darwin, Mott, ... , Figari-Teta, (B)BFF*):



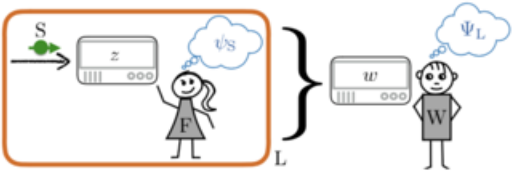
Dark cavity

Illuminated cavity

→ Must understand the special role of light in producing events, facts and "classicality"!

Generalities - 4: The Schrödinger Equation does *Not* Describe the *Time Evolution* of *States* in *QM*

To show this consider real exp. ($\nearrow$ FFS) or/gedanken exp., such as *Wigner's friend paradox* ($\nearrow$ Wigner, Hardy, Frauchiger-Renner, ...):



Courtesy *Frauchiger & Renner*

[*F* measures the spin of the green particle in the *z*-direction. After a successful measurement, but *without knowing* its result, *F* makes predictions of future measnts. using a mixed state, while *W* uses *unitary evolution* of the *pure* initial state of the entire lab, including *F*, to make predictions. The statistics of future measurement outcomes predicted by *F* and *W* are then **different**.

$\Rightarrow$ It is **not** true that the state of the lab evolves in time **linearly !**]

$\rightarrow$ Must find the law of evolution of states in *QM* !

2. The *ETH*-Approach to NR Quantum Mechanics

Purpose: Clarify the notions of *states* in *QM*, of *Potential* & *Actual Events* featured by *isolated open physical systems* $\rightarrow$

$$\boxed{\textit{q.m.}\ \textbf{Law}\ \textit{describing the stochastic evolution of states}}$$

Let S be a physical system. **Physical quantities** characteristic of S, including *potential events* in S, are represented by certain abstract *bounded, selfadjoint linear operators*

$$\widehat{X} = \widehat{X}^* \in \mathcal{O}_S \ \text{(a family of operators)},$$

where the only properties of $\mathcal{O}_S$ are that it contains $\mathbf{1}$ and that if $\widehat{X} \in \mathcal{O}_S$ and F is a bounded continuous function then $F(\widehat{X}) \in \mathcal{O}_S$.

Time is a fundamental quantity in NR physics. It is described by $\mathbb{R}$, or, as in the following, by $\mathbb{Z}$, and parametrizes evolution of S. Associated with every time t there is a representation of $\mathcal{O}_S$ by concrete selfadjoint operators on a separable Hilbert space $\mathcal{H}_S$:

$$\mathcal{O}_S \ni \widehat{X} \mapsto X(t) = X(t)^* \in B(\mathcal{H}_S)\,.$$

Physical Quantities and Their Time Evolution in the Heisenberg Picture

[In concrete examples of physical systems, $X(t)$ can be localized in space and in time – *Haag* talks of "local observables", *Bell* of "local beables":

$$X(t) = \int_I d\tau \int_{\mathbb{R}^3} d\mathbf{x}\, \mathfrak{x}(\tau + t, \mathbf{x})\, h(\tau, \mathbf{x})\,,$$

where $\mathfrak{x}(\tau, \mathbf{x})$ is a hermitian operator-valued distribution on $\mathcal{H}_S$ (e.g., a particle density, a component of a spin density, or an energy density, ...); $h(\tau, \mathbf{x})$ is a real test function with support in a compact interval, I, of the time axis. We then say that $X(t)$ is *localized in the time interval* $I + t$.]

→ Concrete examples motivate our assumption that *every operator* X *representing a physical quantity* $\widehat{X} \in \mathcal{O}_S$ *is localized in a compact interval*, denoted I_X, *of the time axis*; ($I_{F(X)} = I_X$, for F as above).

In the *Heisenberg picture*, time evolution of operators $X(t)$ representing physical quantities $\widehat{X}$ of an *isolated* physical systems S is described by unitary conjugation with the *propagator* of the system (H: Hamiltonian):

Algebras Generated by Operators Representing Physical Quantities Localized in Compact Intervals of Time

$$X(t') = e^{i(t'-t)H/\hbar} X(t) e^{-i(t'-t)H/\hbar}, \qquad \text{for} \quad t, t' \ \text{in} \ \mathbb{R}\,. \tag{1}$$

Let I be an arbitrary interval of *future times*, i.e., $I \subset [t_0, \infty)$, where t_0 is the *present*. We define $\mathcal{E}_I$ to be the *-algebra generated by *arbitrary finite sums of arbitrary finite products of operators*

$$\left\{ X \,\middle|\, X \text{ represents } \widehat{X} \in \mathcal{O}_S \text{ at some time } \geq t_0, \text{ with } I_X \subseteq I \right\}.$$

We define

$$\mathcal{E}_{\geq t} := \overline{\bigvee_{I \subset [t,\infty)} \mathcal{E}_I}, \quad \text{and} \quad \mathcal{E} := \overline{\bigvee_{t \in \mathbb{R}} \mathcal{E}_{\geq t}}^{\|\cdot\|}, \tag{2}$$

where the algebras $\mathcal{E}_{\geq t}, t \in \mathbb{R}$, are assumed to be *weakly* closed![1]
By definition,

$$\mathcal{E}_I \supseteq \mathcal{E}_{I'} \ \text{ if } \ I \supseteq I'\,, \quad \mathcal{E}_{\geq t} \supseteq \mathcal{E}_{\geq t'} \ \text{ if } \ t' > t\,.$$

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[1]Passing to von Neumann algebras is convenient, because the spectral projections of any element of the algebra will then also belong to the algebra!

The Principle of Diminishing Potentialities

Definition 1: Let S be an isolated physical system. *Potential (future) Events* in S that might actualize at a time $t \geq t_0$ are special kinds of physical quantities, namely elements of *partitions of unity*, $\mathfrak{F}_t$,

$$\mathfrak{F}_t = \{\pi_\xi \,|\, \xi \in \mathfrak{X}\} \subset \mathcal{E}_{\geq t}, \quad t \geq t_0, \ \mathfrak{X} \ \text{countable}\ , \sum_{\xi \in \mathfrak{X}} \pi_\xi = \mathbf{1}\,,$$

of *disjoint orthogonal projections*, $\pi_\xi = \pi_\xi^*$ on $\mathcal{H}_S$, with $\pi_\xi \cdot \pi_\eta = \delta_{\xi\eta}\pi_\xi$.

An *isolated* system S is *defined* in terms of a co-filtration $\{\mathcal{E}_{\geq t} \,|\, t \in \mathbb{R}\}$ of algebras generated by *Potential Events* satisfying Eq. (1).

The *Principle of Diminishing Potentialities (PDP)* is the statement that

$$\boxed{\mathcal{E}_{\geq t} \underset{\neq}{\supset} \mathcal{E}_{\geq t'}\ , \quad \text{whenever} \ \ t' > t \geq t_0.} \tag{3}$$

This principle characterizes *isolated open* systems. It will be shown to hold in simple models discussed later in this talk. (*Closed* systems are ones for which $\mathcal{E}_{\geq t} \equiv \mathcal{E}$ is *independent* of t ...)

A **state** of S at time t is given by a *quantum probability measure* on the lattice of orthogonal projections in $\mathcal{E}_{\geq t}$, i.e., a functional, ω_t, with props.:

Potentialities and Actualities ($\nearrow$ Aristotle)

- ω_t assigns to every orthogonal projection $\pi \in \mathcal{E}_{\geq t}$ a non-negative number $\omega_t(\pi) \in [0, 1]$, with $\omega_t(\mathbf{1}) = 1$,
- ω_t is *additive*, in the sense that

$$\sum_{\pi \in \mathfrak{F}_t} \omega_t(\pi) = 1, \quad \forall \text{ partitions of unity } \mathfrak{F}_t \subset \mathcal{E}_{\geq t}\,. \tag{4}$$

Remark: *Gleason*'s theorem (as generalized by *Maeda*) says that states, ω_t, of S at time t, as specified above, are *positive, normal, normalized linear functionals* on $\mathcal{E}_{\geq t}$, i.e., *states* on $\mathcal{E}_{\geq t}$ in the usual sense. (States, ω_t, can be obtained by restricting density matrices, ω, on $\mathcal{H}_S$ to $\mathcal{E}_{\geq t}$.)

Note that ω might be a *pure* "state." But, since $\mathcal{E}_{\geq t} \subsetneq B(\mathcal{H}_S)$, $\forall t < \infty$, assuming that *(PDP)* holds, ω_t will generally be a *mixed* state on $\mathcal{E}_{\geq t}$: *Entanglement!* This observation opens the door towards a natural notion of *Actual Events* – "*actualities*" – in our formalism and to a theory of direct/projective measurements and observations.

In accordance with the *"Copenhagen interpretation"* of *QM*, we say that some *Potential Event* in a partition of unity $\mathfrak{F}_t = \{\pi_\xi | \xi \in \mathfrak{X}\} \subset \mathcal{E}_{\geq t}$ (see Def. 1, last slide) *actually happens* in the interval $[t, \infty)$ of times,

The Centralizer of a State and its Center

i.e., becomes an *Actual Event* setting in at time t, iff

$$\boxed{\omega_t(A) = \sum_{\xi \in \mathfrak{X}} \omega(\pi_\xi \, A \, \pi_\xi), \quad \forall A \in \mathcal{E}_{\geq t}\,,} \tag{5}$$

no off-diagonal elements on R.S. of (5) – *incoherent* superposition![2]

Next, we render the meaning of Eq. (5) precise.

Let $\mathfrak{M}$ be an algebra, and let ω be a state on $\mathfrak{M}$. We define the *centralizer* of a state ω on $\mathfrak{M}$ by

$$\mathcal{C}_\omega(\mathfrak{M}) := \left\{ X \in \mathfrak{M} \,\middle|\, \omega([A, X]) = 0, \ \forall A \in \mathfrak{M} \right\}$$

Note that $\mathcal{C}_\omega(\mathfrak{M})$ is a subalgebra of $\mathfrak{M}$ and that ω is a normalized trace on $\mathcal{C}_\omega(\mathfrak{M})$ ... ! The *center*, $\mathcal{Z}_\omega(\mathfrak{M})$, of $\mathcal{C}_\omega(\mathfrak{M})$ is defined by

$$\mathcal{Z}_\omega(\mathfrak{M}) := \left\{ X \in \mathcal{C}_\omega(\mathfrak{M}) \,\middle|\, [X, A] = 0, \forall A \in \mathcal{C}_\omega(\mathfrak{M}) \right\}. \tag{6}$$

$\rightarrow$ Good general notion of *Actual Events* – "actualities": Let S be an isolated physical system. In (6) we set $\mathfrak{M} := \mathcal{E}_{\geq t}$, $\omega := \omega_t$.

[2] Mathematical precision compells us to suppose that time is discrete.

Actual Events and time evolution of states

Definition 2: Given that ω_t is the state of S at time t, an *Actual Event* is setting in at time t iff $\mathcal{Z}_{\omega_t}(\mathcal{E}_{\geq t})$ contains at least two non-zero orthogonal projections, $\pi^{(1)}, \pi^{(2)}$, which are disjoint, i.e., $\pi^{(1)} \cdot \pi^{(2)} = 0$, and have non-vanishing *"Born probabilities"*, i.e.,

$$0 < \omega_t(\pi^{(i)}) < 1 \,, \qquad \text{for } \; i = 1, 2 \,. \qquad \square$$

Let us suppose for simplicity that $\mathcal{Z}_{\omega_t}(\mathcal{E}_{\geq t})$ is generated by a partition of unity $\mathfrak{F}_t = \{\pi_\xi | \xi \in \mathfrak{X}_{\omega_t}\}$ of orth. proj., where $\mathfrak{X}_{\omega_t} = \text{spec}[\mathcal{Z}_{\omega_t}(\mathcal{E}_{\geq t})]$ is a countable set. Then *Eq. (5) holds true!*

The **Law** describing the *time evolution of states* in *QM* is derived from the following *State Reduction-, or Collapse Postulate*, which makes precise mathematical sense if *time* is **discrete** (!):
Let ω_t be the state of S on $\mathcal{E}_{\geq t}$ at time t. Let dt denote a time step; (dt may be positive if time is discrete; otherwise we will let dt tend to 0 at the end of our constructions).

The State-Reduction (Collapse) Postulate

We define a state on the algebra $\mathcal{E}_{\geq t+dt}$ by setting

$$\overline{\omega}_{t+dt} := \omega_t\big|_{\mathcal{E}_{\geq t+dt}} .$$

Axiom CP: *Let $\mathfrak{F}_{t+dt} := \left\{\pi_\xi \,|\, \xi \in \mathfrak{X}_{\overline{\omega}_{t+dt}}\right\}$ be the partition of unity generating the spectrum, $\mathfrak{X}_{\overline{\omega}_{t+dt}}$, of $\mathcal{Z}_{\overline{\omega}_{t+dt}}(\mathcal{E}_{\geq t+dt})$.*

Then **'Nature'** *replaces the state $\overline{\omega}_{t+dt}$ on $\mathcal{E}_{\geq t+dt}$ by a state*

$$\omega_{t+dt}(\cdot) \equiv \omega_{t+dt,\xi}(\cdot) := \overline{\omega}_{t+dt}(\pi_\xi)^{-1} \cdot \overline{\omega}_{t+dt}(\pi_\xi(\cdot)\pi_\xi) ,$$

for some $\xi \in \mathfrak{X}_{\omega_{t+dt}}$, with $\omega_{t+dt}(\pi_\xi) \neq 0$.

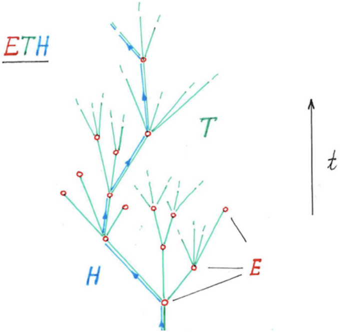
The probability, $prob_{t+dt}(\xi)$, for the state $\omega_{t+dt,\xi}$ to be selected by Nature as the state of S at time $t+dt$ is given by

$$prob_{t+dt}(\xi) = \overline{\omega}_{t+dt}(\pi_\xi) \qquad (Born's\ Rule) \qquad \square \tag{7}$$

<u>*Remark*</u>*:* The mathematical theory obtained when the time step, dt, tends to 0 is not analyzed rigorously, yet. $\rightarrow$ Challenge for math.!

A Metaphoric Picture of the Time Evolution of States in QM – According to "*ETH*"

Apparently, the time-evolution of *states* of a phys. system is described by a *stochastic branching process*, with branching rules det. by **Axiom CP**. This can be made precise, mathematically, if time is discrete, and it leads to a good notion of *projective measurements*; see models discussed later.



Rudolf Haag

E: "Events", *T*: "Trees" of possible states, *H*: "Histories" of states.

This is different from and supercedes the "decoherence mumbo-jumbo"!

Remarks on the *ETH*-Approach

1. *Actual Events* might be recorded by *"projective measurements"* of *physical quantities*: If an operator $X \in \mathcal{E}_{\geq t}$ repr. a physical quantity of S is "well approximated" ... by an operator in $\mathcal{Z}_{\omega_t}(\mathcal{E}_{\geq t})$ then the *actual event* setting in at time t amounts to a measurement of X.

2. A *passive state*, ω_t, is a state for which $\mathcal{Z}_{\omega_t}(\mathcal{E}_{\geq t})$ consists of only two projections, π and $\pi^{\perp}$, with $\omega_t(\pi) = 1$ and $\omega_t(\pi^{\perp}) = 0$. If ω_t is passive it does not feature any event at time t. If ω_t is time- transl. invariant & passive then $\omega_{t'}$ is passive, $\forall t' > t$. *Thermal equilibrium states* and states of *closed systems* are passive at all times.

3. A microscopic system only **weakly** coupled to the radiation field has the property that, for most times t, $\mathcal{Z}_{\omega_t}(\mathcal{E}_{\geq t})$ contains a projection, π_0, with the property that $\omega_t(\pi_0) \simeq 1$, while $\omega_t(\pi) \simeq 0, \forall \pi \neq \pi_0$ in $\mathcal{E}_{\geq t}$. The state of such a system is nearly *constant* in time, in the Heisenberg picture (i.e., evolves according to the *Schrödinger eq.* in the Schrödinger picture), *except for rare instances* when an unlikely event makes it jump. For purely *entropic reasons*, such rare jumps occur at a possibly very small, but *non-zero* rate, unless the state of the system is a time-translation invariant passive state.

ETH-Approach to Local Relativistic Quantum Theory

Concerning this topic, I remark that the relativistic theory is based on the following kinds of postulates:

- With every actual event, π, one can associate a C^*-algebra, $\mathcal{E}_{>\pi}$, of *potentialities* that *"lie in the future of* π*"*, with $\pi \notin \mathcal{E}_{>\pi}$.
- The *past* of π consists of *all actual events*, $\pi_- \neq \pi$, with the property that $\mathcal{E}_{>\pi_-}$ contains π and $\mathcal{E}_{>\pi}$; $\overline{\mathcal{E}}_{>\pi}$ is the smallest algebra $\mathcal{E}_{>\pi_-}$, with π_- in the past of π.
- An appropriate modified version of the *Principle of Diminishing Potentialities* then says that if π' is in the past of π then

$$\left(\mathcal{E}_{>\pi}\right)' \cap \mathcal{E}_{>\pi_-} \text{ is non-trivial } (\infty\text{-dim.})\,.$$

- Locality: Every projection σ **not** belonging to $\overline{\mathcal{E}}_{>\pi}$ and **not** in the past of π commutes with π: $[\sigma, \pi] = 0$.
- One can then formulate what is meant by a potential event $\pi_+ (\neq \pi)$ contained in $\mathcal{E}_{\geq\pi}$ (i.e., in the future of π) to *actualize*. It turns out that this can be decided by only knowing π and *all actual events* in the past of π (but in the future of the state the system has been prepared in) ...

Probabilities of histories, given an initial state

One can then introduce history operators: Let $\pi_1, \pi_2, \ldots, \pi_n$ be all the events in the past of π, but in the future of the state ω the system has been repared in, *ordered according to their occurrence.* Thanks to our assumption of *locality*, the following *"history operator"* is well defined:

$$H(n) := \prod_{j=1,\ldots,n}^{\rightarrow} \pi_j \,.$$

The *probability* of the *"history of events"* $\pi_1, \pi_2, \ldots, \pi_n, \pi$, given that the system has been prepared in the state ω in the past of all these events, is given by

$$prob\big\{\pi_1, \pi_2, \ldots, \pi_n, \pi \,|\, \omega\big\} = \omega\big(H(n)\, \pi\, H(n)^*\big) \,.$$

Discussion: ...

The details – not discussed today – can be understood by invoking old results of L. Landau (strict locality) and D. Buchholz (an analysis of Huygens' Principle in QFT). $\rightarrow$

3. Huygens' Principle and *PDP*

This discussion is inspired by *ETH*-Approach to relativistic *QT*.

Huygens' Principle for massless modes (photons, gravitons, ...) in isolated physical systems

$\Rightarrow$ *Principle of Diminishing Potentialities !*

S : Isolated system consisting, for example, of a static atom located at $\mathbf{x} = 0$, coupled to the *electromagnetic field.* Concretely:

- Atom has M internal energy levels, Hilbert space $\mathfrak{h}_A \simeq \mathbb{C}^M$.
- Hilbert space of e.m. field is *Fock space*, $\mathfrak{F}$, of photons; e.m. field described by field tensor, $F_{\mu\nu}(\tau, \mathbf{x})$, with property that, for real-valued test functions $\{h^{\mu\nu}\}$ on space-time, the operator

$$F(h) := \int_{\mathbb{R}\times\mathbb{R}^3} d\tau\, d\mathbf{x}\, F_{\mu\nu}(\tau, \mathbf{x})\, h^{\mu\nu}(\tau, \mathbf{x})$$

is self-adjoint on $\mathfrak{F}$ and satisfies locality. The usual Hamiltonian of the free e.m. field is denoted by H_0; with $H_0 = H_0^* \geq 0$ on $\mathfrak{F}$. Hilbert space of *S*:

$$\mathcal{H}_S := \mathfrak{F} \otimes \mathfrak{h}_A \,.$$

A Concrete Model

Let $V_t^{\pm}$ be the forward/backward light cone above the space-time point $(t, \mathbf{x} = 0)$. We define

Space-time diamonds: $\quad D_{[t,t']} := V_t^+ \cap V_{t'}^-$, with $t' > t$.

Bounded functions of field operators $F(h)$, $\mathrm{supp}(h^{\mu\nu}) \subseteq D_{[t,t']}$, generate a (von Neumann) algebra $\mathcal{A}_{I=[t,t']}$. We then define

$$\mathcal{D}_I^{(0)} := \mathcal{A}_I \otimes \mathbf{1}\big|_{\mathfrak{h}_A}, \qquad \mathcal{E}_I^{(0)} := \mathcal{A}_I \otimes B(\mathfrak{h}_A),$$
$$\mathcal{E}_{\geq t}^{(0)} := \overline{\bigvee_{I \subset [t,\infty)} \mathcal{E}_I^{(0)}}. \tag{8}$$

PDP holds for non-interacting system: Setting $I := [t, t']$, one has that

$$\boxed{\left[\mathcal{E}_{\geq t'}^{(0)}\right]' \cap \mathcal{E}_{\geq t}^{(0)} = \mathcal{D}_I^{(0)} \quad (\text{an } \infty - \text{dim. algebra!})} \tag{9}$$

Remark: Follows from *Huygens' Principle*, namely from the fact that

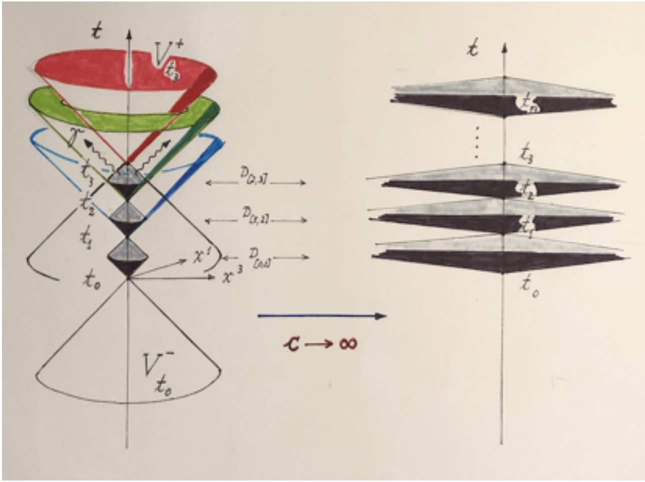
$$[F_{\mu\nu}(x), F_{\rho\sigma}(y)] = 0, \quad \text{unless } x - y \text{ is light-like.}$$

From now on, we discretize time: $t_n := n \in \mathbb{Z}$. Speed of light: c.

Interacting Propagator, Γ / Illustration of *HP*

To describe interactions, pick a unitary operator $U \in \mathcal{E}^{(0)}_{[0,1]}$, and define $\Gamma := e^{-iH_0}U$. Then the propagator of the coupled system is given by

$$\Gamma^n = e^{-inH_0}U(n),\ (\Gamma^n)^* = \Gamma^{-n},\ U(n) = \cdots,\ n = 0, 1, 2, \ldots \tag{10}$$



PDP for the Interacting Model

It suffices to consider time evolution for times $t \geq t_0 := 0$. We define

$$\mathcal{E} := \mathcal{E}^{(0)}_{\geq 0}, \quad \mathcal{E}_{\geq n} := \left\{ \Gamma^{-n} X \Gamma^{n} \,\middle|\, X \in \mathcal{E} \right\}. \tag{11}$$

It is straightforward to verify *PDP* for the *interacting model*: Using (10) and (11), one shows that

$$\left[\mathcal{E}_{\geq n'}\right]' \cap \mathcal{E}_{\geq n} \simeq \mathcal{D}_{[n,n']}\,, \ \text{ for } \ n' > n, \tag{12}$$

where $\mathcal{D}_{[n,n']} := \left\{ U(n')^* X\, U(n') \,\middle|\, X \in \mathcal{D}^{(0)}_{[n,n']} \right\}$. Note that

$$\boxed{\Gamma^{n-n'}\, \mathcal{E}_{\geq n}\, \Gamma^{n'-n} = \mathcal{E}_{\geq n'} \subsetneq \mathcal{E}_{\geq n}\,, \quad \text{for } \ n' > n.}$$

Preparing the system in an initial state, ω_0, at time $n = 0$, we would like to determine the *stochastic time evolution* of states, ω_t, predicted by the law encoded in *Definition 2* (Actual Events) and **Axiom CP** of Sect. 2.

Stochastic Time Evolution with Memory

We consider an *Example:*

$$\omega_0(X) := \mathrm{tr}_{\mathcal{H}_S}\big(\big[|0\rangle\langle 0| \otimes \Omega\big]\, X\big), \quad X \in \mathcal{E}\,,$$

with $|0\rangle$ the *vacuum vector* in $\mathfrak{F}$, and Ω a density matrix on $\mathfrak{h}_A$.

The state ω_0 does *not* entangle the atom with the e.m. field. Yet, interactions *will* entangle them in the course of time, as expected. Since the vacuum $|0\rangle\langle 0|$ is not a *"product state,"* *stochastic time evolution* of states of S exhibits *memory effects* – explicit control rather difficult!

Matters simplify drastically in the limit where the speed of light, c, tends to ∞, which we consider next.

In the limit $c \to \infty$, the regions $D_{[k,k+1]}$ approach **time slices**, $k \leq t \leq k+1$ (Fig.!), and the algebras $\mathcal{D}^{(0)}_{[k,k+1]}$ are given by

$$\mathcal{D}^{(0)}_{[k,k+1]} \simeq B(\mathcal{H}_k), \;\; \mathcal{H}_k \overset{e.g.}{=} \mathbb{C}^N, \;\text{for some}\;\; N \leq \infty\,, \forall\, k. \qquad (13)$$

4. A Heavy Atom Interacting with the R-Field

with *Alessandro Pizzo*

As $c \to \infty$: electromagnetic field $\to$ "R-field" ($N < \infty$, henceforth).

Follow evolution of S only for $t \geq 0$. Pick an orthonormal basis $\{\phi_j\}_{j=0}^{N-1}$ in $\mathbb{C}^N$; $\mathcal{S}_{fin} :=$ set of sequences $\underline{k} := \{k_n\}_{n=0}^{\infty}$, with $k_n = 0, \ldots, N-1$ and $k_n = 0$, except for *finitely many* values of $n \in \mathbb{Z}_+$. For $\underline{k} \in \mathcal{S}_{fin}$, we define

$$\Phi_{\underline{k}} := \bigotimes_{n=0}^{\infty} \phi_{k_n}\,, \qquad \Phi_{\underline{0}} : \text{"}\textit{vacuum}\text{" (reference vector).} \tag{14}$$

The Hilbert space, $\mathfrak{F}_{\underline{0}}$, of the R-field is then given by the closure of the space of finite linear combinations of vectors $\left\{\Phi_{\underline{k}} \,\middle|\, \underline{k} \in \mathcal{S}_{fin}\right\}$ in the norm determined by the scalar product defined by

$$\langle \Phi_{\underline{k}}, \Phi_{\underline{k'}} \rangle := \prod_{n=0}^{\infty} \delta_{k_n, k'_n}\,. \tag{15}$$

We then set

$$\mathcal{H}_S := \mathfrak{F}_{\underline{0}} \otimes \mathfrak{h}_A\,.$$

The Propagator of the Model

We define a *shift*, σ, on $\mathcal{S}_{fin}$ by $\sigma(\underline{k})_n := k_{n+1}$, and define the *time-1 propagator of the R-field* (before coupling to A) by

$$\mathfrak{S}\Phi_{\underline{k}} := \Phi_{\sigma(\underline{k})}, \quad \underline{k} \in \mathcal{S}_{fin} \,,$$

extended to $\mathfrak{F}_{\underline{0}}$ by linearity. Note that $\mathfrak{S}\Phi_{\underline{0}} = \Phi_{\underline{0}}$. The *time-1 propagator of the atom* (before couplng to R-field) is given by a unitary operator V on $\mathfrak{h}_A$, and we set $\Gamma_0 := \mathfrak{S} \otimes V$.

To introduce *interactions*, pick unitary U on $\mathbb{C}^N \otimes \mathfrak{h}_A$ and define

$$\begin{aligned} U_1 := U\big|_{\mathcal{H}_0}, U_k := \Gamma_0^{1-k} U_1 \Gamma_0^{k-1},\ k = 1, 2, \dots \,, \\ U(n) := U_n \dots U_1,\ n = 1, 2, \dots \,, \end{aligned} \tag{16}$$

Interacting propagator of model given by $\{\Gamma^n\}_{n=0,1,2,\dots}$, where

$$\Gamma := \Gamma_0 U_1 \ \text{(unitary)} \ \Rightarrow \ \Gamma^n = \Gamma_0^n U(n),\ \forall\, n \in \mathbb{Z}_+ \,, \tag{17}$$

Time Evolution of States, According to "*ETH*"

Algebras:

$$\mathcal{E} := \overline{\left\{\text{finite sums of ops. } \; F \otimes C \,\middle|\, F \in B(\mathfrak{F}_{\underline{0}}), C \in B(\mathfrak{h}_A)\right\}},$$

$$\mathcal{E}_{\geq n} := \left\{\Gamma^{-n} X \, \Gamma^n \,\middle|\, X \in \mathcal{E}\right\}, \quad n = 0, 1, 2, \dots \tag{18}$$

<u>Initial state</u>: Let Ω_0 be a density matrix on $\mathfrak{h}_A$. We set

$$\omega_0(X) := \langle \Phi_{\underline{k}}, F\,\Phi_{\underline{k}} \rangle \cdot \mathrm{tr}\big(\Omega_0 \cdot C\big), \text{ with } \underline{k} \in \mathcal{S}_{fin}, \; X = F \otimes C \in \mathcal{E}.$$

Our aim is to determine the time evolution of ω_0 according to the Law ($\nearrow$ *Definition 2* & **Axiom CP**) of the *ETH*-Approach. Using *induction in time* n, we then find that state, ω_n, on $\mathcal{E}_{\geq n}$ is given by

$$\omega_n\big(\Gamma^{-n} X \, \Gamma^n\big) = \langle \Phi_{\sigma^n(\underline{k})}, F\,\Phi_{\sigma^n(\underline{k})} \rangle \cdot \mathrm{tr}\big(\Omega_n \cdot C\big), \tag{19}$$

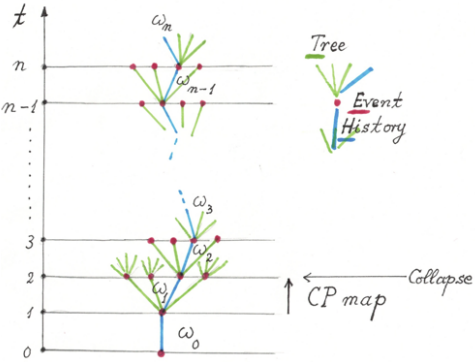
where Ω_n is a density matrix on $\mathfrak{h}_A \propto$ an orthogonal projection; $\big\{\Omega_n\big\}_{n=0,1,2,\dots}$: sample path of a *stochastic branching process*:

Time Evolution of States – Summary

The *stochastic time evolution* of states in our model,

$$\omega_0 \to \cdots \to \omega_{n-1} \to \omega_n \to \cdots, \qquad \omega_0 \text{ as above,}$$

is described in terms of a *quantum Markov chain* which depends on $\underline{k} \in \mathcal{S}_{fin}$ and acts on density matrices of atom. The sample paths, $\{\Omega_n\}_{n=0}^{\infty}$, are obtained by *"unravelling"* this Markov chain; (next slide).



A Very Simple Explicit Model

A simple example of an operator U describing interactions "$A - R$":

Let $\{Q_m\}_{m=1}^{M}$ be a partition of unity by orthogonal projections on $\mathfrak{h}_A$ – for ultimate simplicity, $Q_m = |\psi_m\rangle\langle\psi_m|$, where $\{\psi_m\}_{m=1}^{M}$ is a CONS. Let $T^{(m)}$ be a unitary operator on $\mathbb{C}^N$, $\forall\, m = 1, \dots, M$. We define

$$U := \sum_{m=1}^{M} T^{(m)} \otimes Q_m \,.$$

We follow *stochastic evolution* of initial state ω_0 according to *ETH*. It turns out that if ω_0 is chosen as above then, after $n = 1, 2, \dots$ time steps, the formula for the state ω_n, applied to operators of the form $\Gamma^{-n}(F \otimes C)\Gamma^n \in \mathcal{E}_{\geq n}$, is given by

$$\boxed{(\mathcal{I}_n) \qquad \omega_n\big(\Gamma^{-n}(F \otimes C)\Gamma^n\big) = \langle \Phi_{\sigma^n(\underline{k})}, F\, \Phi_{\sigma^n(\underline{k})}\rangle \cdot \mathrm{tr}\big(\Omega_n \cdot C\big)} \qquad (20)$$

where $\Omega_n \propto$ orthogonal projection.

We now explain the *induction step* $(\mathcal{I}_n) \Rightarrow (\mathcal{I}_{n+1})$. We first consider the restriction of ω_n to the algebra $\mathcal{E}_{\geq(n+1)}$:

$$\omega_n\big(\underbrace{\Gamma^{-(n+1)}(F \otimes C)\Gamma^{n+1}}_{\equiv X \in \mathcal{E}_{\geq(n+1)}}\big) = \langle \Phi_{\sigma^{n+1}(\underline{k})}, F\, \Phi_{\sigma^{n+1}(\underline{k})}\rangle \cdot \mathrm{tr}\big(\widehat{\Omega}_{n+1} \cdot C\big),$$

The Induction Step

where the density matrix $\widehat{\Omega}_{n+1}$ is given by

$$\widehat{\Omega}_{n+1} = \sum_{\ell,m=1,\dots,M} g^{m\ell}(n)\, VQ_\ell\, \Omega_n\, Q_m V^*\,, \tag{21}$$

$$V \text{ unitary on } \mathfrak{h}_A\,, \qquad g^{m\ell}(n) := \langle T^{(m)}\phi_{k_n}, T^{(\ell)}\phi_{k_n}\rangle \tag{22}$$

$G(n) := \big(g^{m\ell}(n)\big)$ is a non-negative matrix. Map $\Omega_n \mapsto \widehat{\Omega}_{n+1}$ given in (21) is *completely positive*. $\Rightarrow$ $\widehat{\Omega}_{n+1}$ is a density matrix. Spect. thm. $\Rightarrow$

$$\widehat{\Omega}_{n+1} = \sum_{j=1}^{L} p_j(n+1)\,\Pi_j(n+1), \quad p_1(n+1) > \cdots > p_L(n+1) > 0,$$

for some $L \leq M$, where the $\Pi_j(n+1)$ are orthogonal projections, and

$$\sum_{j=1}^{L} p_j(n+1)\,\mathrm{tr}\big(\Pi_j(n+1)\big) = 1\,.$$

According to the *Collapse Postulate*, **Axiom CP**, Nature chooses

The Weak-Coupling Regime

$$\Omega_{n+1} := \left[\mathrm{tr}\left(\Pi_{j*}\right)\right]^{-1}\Pi_{j*}(n+1)\,, \quad \text{for some } \; j_*\,, \tag{23}$$

as the state of the atom at time $n+1$, with

$$\text{Probablity} = p_{j*}(n+1)\mathrm{tr}\left(\Pi_{j*}(n+1)\right) \quad \text{(Born Rule)}$$

This proves $(\mathcal{I}_{n+1})$, thus completing the induction step.

The weak-coupling regime: $T^{(m)} = \mathbf{1} + \varepsilon\,\tau^{(m)}, \quad \|\tau^{(m)}\| \leq 1,$
for some positive $\varepsilon \ll 1$. Then

$$g^{m\ell}(n) = 1 + \mathcal{O}(\varepsilon), \qquad \forall\; m, \ell.$$

Thus Eq. (21) implies

$$\widehat{\Omega}_{n+1} = V\,\Omega_n\,V^* + \mathcal{O}(\varepsilon) \;\Rightarrow\; \Omega_{n+1} = V\,\Omega_n\,V^* + \mathcal{O}(\varepsilon)\,, \tag{24}$$

with probability $1 - \mathcal{O}(\varepsilon)$, i.e., *time evolution of states* (in the Schrödinger picture) *is given, to a good approximation, by unitary conjugation!*

However, for *purely entropic reasons*, it happens with a frequency $\propto \varepsilon$ that $\mathrm{tr}(\Omega_{n+1} \cdot V\Omega_n V^*) \sim 0$. This is then perceived as an *"Event"* in the literal sense of the word!

The Strong-Coupling Regime

The strong-coupling regime: Characterized by

$$g^{m\ell}(n) = \langle T^{(m)}\,\phi_{k_n},\, T^{(\ell)}\,\phi_{k_n}\rangle = \delta_{m\ell} + \mathcal{O}(\varepsilon), \quad 0 < \varepsilon \ll 1\,, \tag{25}$$

(at least for $k_n = 0$!) Then, for large enough times, n,

$$\widehat{\Omega}_{n+1} = \sum_{m=1}^{M} VQ_m\,\Omega_n\,Q_m V^* + \mathcal{O}(\varepsilon)\,, \tag{26}$$

hence $\Omega_{n+1} = VQ_k V^* + \mathcal{O}(\varepsilon)$, for some $k \in \{1, \ldots, M\}$ (state collapse!). If $\Omega_n = V\,Q_\ell V^* + \mathcal{O}(\varepsilon)$ then the probability for Ω_{n+1} to be given by $\Omega_{n+1} = VQ_k V^* + \mathcal{O}(\varepsilon)$ is given by

$$P(k,\ell) := \mathrm{tr}\Big(\big|Q_k V\,Q_\ell\big|^2\Big)\,. \tag{27}$$

Hence the evolution of states is well approximated by sample paths of a *classical Markov chain* with transition function $P(k,\ell)$!

Alternation Between Unitary Evolution and State Collapse

It can happen that the matrices $G(n) = \big(g^{m\ell}(n)\big)$ have the form $G(n) = G_0 + \mathcal{O}(\varepsilon)$, with

$$G_0 = \begin{pmatrix} 1 & \dots & 1 & 0 & \dots & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ 1 & \dots & 1 & 0 & \dots & 0 \\ 0 & \dots & 0 & 1 & \dots & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ 0 & \dots & 0 & 0 & \dots & 1 \end{pmatrix} \tag{28}$$

where the upper left block is a $K \times K$ matrix and the lower right block is the $(M-K) \times (M-K)$ identity matrix. Then unitary evolution prevails on the subspace $\mathfrak{h}_A^w$ of dimension K corresp. to the range of the proj. $\sum_{m=1}^{K} Q_m$, while on the complementary subspace $\mathfrak{h}_A^s = \mathfrak{h}_A \ominus \mathfrak{h}_A^w$ state collapse prevails. If the subsapces $\mathfrak{h}_A^w$ and $\mathfrak{h}_A^s$ are not invariant under V then there are transitions frrom one regime to the other regime in the course of the evolution of a state.

This leads to a succinct description of *measurements and observations*!

Summary and Conclusions ...

- The *ETH-Approach to Quantum Mechanics* provides a logically coherent theory of *Potential and Actual Events*, of the recordings of the latter, and of measurements. It has resemblences with "Many Worlds," "GRW," ... ; yet, it supersedes these imprecise formalisms and describes but **One World**! The models in Sect. 4 provide a useful illustration of the *ETH*-Approach.
- As in the genesis of Special Relativity, *fields describing massless modes* (photons & gravitons), besides the even-dimensionality of space-time play key roles in the genesis of a Quantum Theory that satisfies the spectrum condition ($H \geq 0$) and solves the "measurement problem." (Has not been properly appreciated, so far!)
- *Actual Events* weave the fabric of space-time! ("Emergent gravity")
- Thanks to the *Principle of Diminishing Potentialities* (*PDP*) and the natural presence of an "arrow of time" in the *ETH*-Approach to Quantum Theory, the "Information –" and the "Unitarity Paradox" appear to dissolve. ...

I thank you for your attention!